

Electromagnetic Memory

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Abstract

An elementary derivation of the electromagnetic memory effect is given. An experimental setup to detect it is suggested.

Classical memory effects and their relation to BMS conservation laws and soft emission theorems have been the subject of recent interest by Strominger and collaborators (see [1] and references contained therein.) In this note I will give an elementary derivation of the electromagnetic memory effect and suggest a way of detecting it.

1 Memory Effect

Consider a large sphere Ω surrounding an explosion which ejects charged particles, which later pass through the sphere. We assume the explosion is near the center of the sphere so that the particles' velocities are radial when they pass through the sphere. We also assume that they move with velocity close to or at the speed of light. Before the explosion the charge density, current density and electromagnetic field were zero. We work in the temporal gauge and take the initial value of the vector potential to be $A_{in} = 0$.

The Gauss equation

$$\nabla \cdot E = \rho \tag{1.1}$$

is true everywhere at all time. We will consider it on the sphere.

Since the charges are moving in a lightlike radial trajectory when they pass the sphere we can assume that the charge density is equal to the radial component of the current j_r . Thus, on the sphere we may write,

$$\nabla \cdot \dot{A} = -j_r \tag{1.2}$$

where we have used $E = -\dot{A}$.

Now integrate over time and we find (on the sphere) that after all charges have passed through the sphere,

$$\nabla \cdot A = -Q(\Omega, \infty) \tag{1.3}$$

where $Q(\Omega, \infty)$ is the total charge that has passed through the point Ω after all particles have left.

It is easy to show that the contribution of the normal components of E average to zero

as the charges pass through the surface of the sphere. As the charge recedes from the surface the normal component of E is opposite to the value it had while the charge was approaching the surface.

Therefore we may restrict the divergence of \dot{A} to the components along the sphere. The subscript Ω indicates the restriction to the sphere. Thus at the end of the process we find that at every point on the sphere:

$$\nabla_{\Omega} \cdot A_{\Omega} = -Q(\Omega, \infty) \quad (1.4)$$

Now let us imagine that the sphere is covered with a collection of superconducting nodes. Initially before the explosion the superconducting nodes are connected by wires with resistors, until all currents decay to zero. At that point we can be sure that the relative phases of the superconducting condensates are all zero. Then we disconnect the wires.

At the end of the experiment there is a gauge field A present on the sphere but no electric or magnetic field. Therefore the gauge field has the form,

$$A = \nabla \lambda. \quad (1.5)$$

We can eliminate the gauge field by a gauge transformation on the sphere at the cost of creating a relative phase between the superconduction nodes. The position dependent phase is just λ . This frozen-in phase is the electromagnetic memory effect. It can be detected by reconnecting the nodes with wires. Currents will flow proportional to the phase differences.

2 Local Conservation Law

We can express the memory effect as an instantaneous conservation law. Define $Q(\Omega, t)$ to be the total charge that has passed through the point Ω up to time t . Obviously

$$\dot{Q}(\Omega, t) = j_r(\Omega, t) \quad (2.1)$$

The Gauss condition becomes

$$\frac{d}{dt} \{ \nabla_{\Omega} \cdot A_{\Omega} + Q(\Omega, t) \} = 0 \quad (2.2)$$

Equation 2.2 is a conservation law that is true at every point on Ω . The reason that it is not trivial is that when integrated over time the change in A is not zero since it must satisfy 1.4. As we have seen this leads to an observable flow of charge between superconductors. The flow will occur when we reconnect the nodes no matter how long we wait. The memory of the explosion is frozen into the relative phases.

A last point is that the integrated conservation law may also be understood as the usual soft photon emission theorem. Thus we see the triangle of ideas: Local conservation law, soft theorem, memory effect.

3 Generalization

The motion of charges does not have to be light-like to have a memory effect although the analysis is not as elegant. For simplicity assume the charges move radially but this is not essential. Also assume that the charge density and electromagnetic fields are all zero inside the sphere at the beginning and end of the process.

The simplest case is when the charges move with fixed velocity $v < 1$. In that case we can write

$$j_r = v\rho \quad (3.1)$$

and replace 1.2 by

$$\nabla \cdot \dot{A} = -\frac{1}{v} j_r \quad (3.2)$$

and 1.3 by

$$\nabla \cdot A = -\frac{1}{v} Q(\Omega, \infty) \quad (3.3)$$

If the velocity at the sphere is time dependent then 3.3 becomes more complicated with the right side being an integral.

Another case is when the charges never pass through the sphere but fall back to the center and annihilate. In that case we write

$$\nabla \cdot E = 0 \tag{3.4}$$

on the sphere, and break up the divergence into a spherical and radial part.

$$\nabla_{\Omega} \cdot \dot{A}_{\Omega} = \frac{\partial E_r}{\partial r}. \tag{3.5}$$

Integrating, the final A_{Ω} satisfies:

$$\nabla_{\Omega} \cdot A_{\Omega} = \int dt \frac{\partial E_r}{\partial r}. \tag{3.6}$$

There is no reason for this to be zero. For example if the charge distribution is dipole with a positive dipole moment at all time then $\nabla_{\Omega} \cdot A_{\Omega}$ should have a positive dipole moment.

4 Remarks

- Since in all cases it is assumed that the magnetic field is zero and the end of the process,

$$\nabla \times A = 0 \tag{4.1}$$

on the sphere. In particular the two-dimensional curl on the sphere vanishes. Therefore A is completely determined by a two dimensional potential problem with $\nabla_{\Omega} \cdot A_{\Omega}$ playing the role of the charge density.

- The case with light-like velocity is easy to think of in terms of a conservation law and presumably a BMS type symmetry. It's not clear to me what the conserva-

tion/symmetry is for the more general case.

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References

- [1] D. Kapec, M. Pate and A. Strominger, “New Symmetries of QED,” arXiv:1506.02906 [hep-th].